Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS
UNIT Methods 1 & 2

Section One:
Calculator-free

Name: _____ Soln _____

Teacher's Name: _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Mark	Question	Mark
1		6	
2		7	
3		8	
4		9	
5		Total	/52

MATHEMATICS METHODS 1 & 2   CALCULATOR-FREE

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	9	9	50	**52**	35
Section Two: Calculator-assumed	14	14	100	95	65
				Total	100

Instructions to candidates

1.   The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer booklet.

3.   You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4.   Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5.   **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6.   It is recommended that you **do not use pencil**, except in diagrams.

7.   The Formula sheet is **not** to be handed in with your Question/Answer booklet.

Section One: Calculator-free   **(52 Marks)**

This section has **nine (9)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1   **(5 marks)**

Solve the following

(a) $\frac{5x}{6} = \frac{7}{2}$ (1 mark)

$$10x = 42$$

$$\therefore x = 4.2 \quad\checkmark \quad = \frac{42}{10} = \frac{21}{5} = 4\frac{1}{5}$$

(b) $\frac{3x}{5} - \frac{x}{4} = 2$ (2 marks)

$$4(3x) - 5(x) = 40 \quad\checkmark$$

$$12x - 5x = 40$$

$$7x = 40$$

$$x = \frac{40}{7} \quad\checkmark \quad \text{or} \quad 5\frac{5}{7}$$

(c) $x^3 + 3x^2 - 4x = 0$ (2 marks)

$$x(x^2 + 3x - 4) = 0$$

$$x(x+4)(x-1) = 0 \quad\checkmark$$

$$\therefore \underline{x = 0}, \quad \underline{x = -4} \quad \text{or} \quad \underline{x = 1} \quad\checkmark$$

Not to Scale

MATHEMA_CS METHODS 1 & 2   CALCULATOR-FREE

Question 2
(12 marks)

A brother and sister are out playing fetch with their dog on a straight slope. The three are spread out evenly with the dog being directly (half-way) in between the brother and the sister.

When modelled on a cartesian plane, the dog's position can be considered as $(7.5, 2.5)$ and the sister's position can be considered as $(10, 3)$

Dog   Ryanne
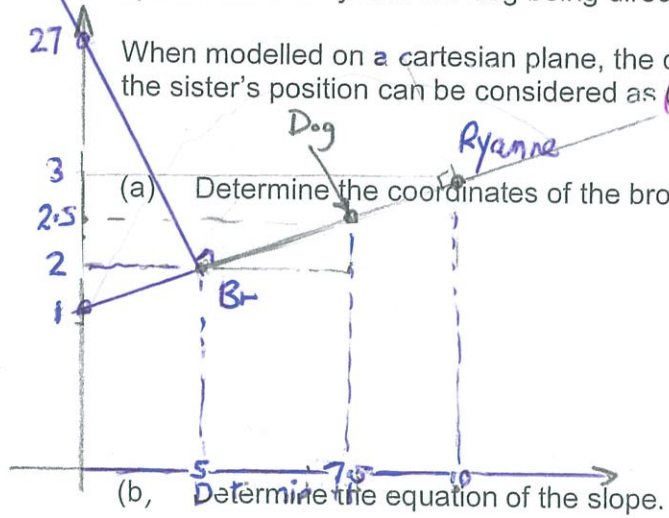
(a)   Determine the coordinates of the brother.   (2 marks)

$$B_r = (7.5 - 2.5, \quad 2.5 - 0.5)$$

$$= (5, 2)$$

(b,   Determine the equation of the slope.   (3 marks)

$$m_1 = \frac{3-2}{10-5} = \frac{1}{5} \checkmark$$

$$\therefore y_1 = \frac{1}{5}x + c \text{ at } (5,2) \quad 2 = \frac{1}{5} \times 5 + c, \; c = 1$$

$$\therefore y_1 = \frac{1}{5}x + 1$$

(c)   The brother is flying a kite. If the line of the kite is running perpendicular to the slope, determine the equation that models the line.   (3 marks)

$$m_1 \times m_2 = -1 \qquad \therefore m_2 = -5 \checkmark$$

$$y_2 = -5x + c \text{ at } (5,2)$$

$$2 = -5 \times 5 + c$$

$$27 = c \checkmark \qquad \therefore y_2 = -5x + 27 \checkmark$$

(d)   The sister throws the ball away from the dog and the path of the ball can be modelled by the following quadratic equation.

−1 if not given as a coord

$$y = -\frac{x^2}{2} + x + 1$$

Determine the coordinates that the ball first lands on the slope.   (4 marks)

$$-0.5x^2 + x + 1 = 0.2x + 1 \checkmark$$

i.e $-0.5x^2 + 0.8x = 0$ $\times$ by -2 and $y = 0.2 \times 1.6 + 1$

$$x^2 - 1.6x = 0$$ $= 1.32 \checkmark = \left(\frac{33}{25}\right)$

$$x(x - 1.6) = 0 \quad \text{or} \left(\frac{8}{5}\right)$$ $\therefore$ The coords is $(1.6, 1.32)$

$$\therefore x = 0 \text{ or } x = 1.6 \checkmark$$

Also $(0,1)$ $\left(\frac{8}{5}, \frac{33}{25}\right)$

Ball (oops)

$(\frac{8}{5}, \frac{33}{25})$

Br

Dog

Ry
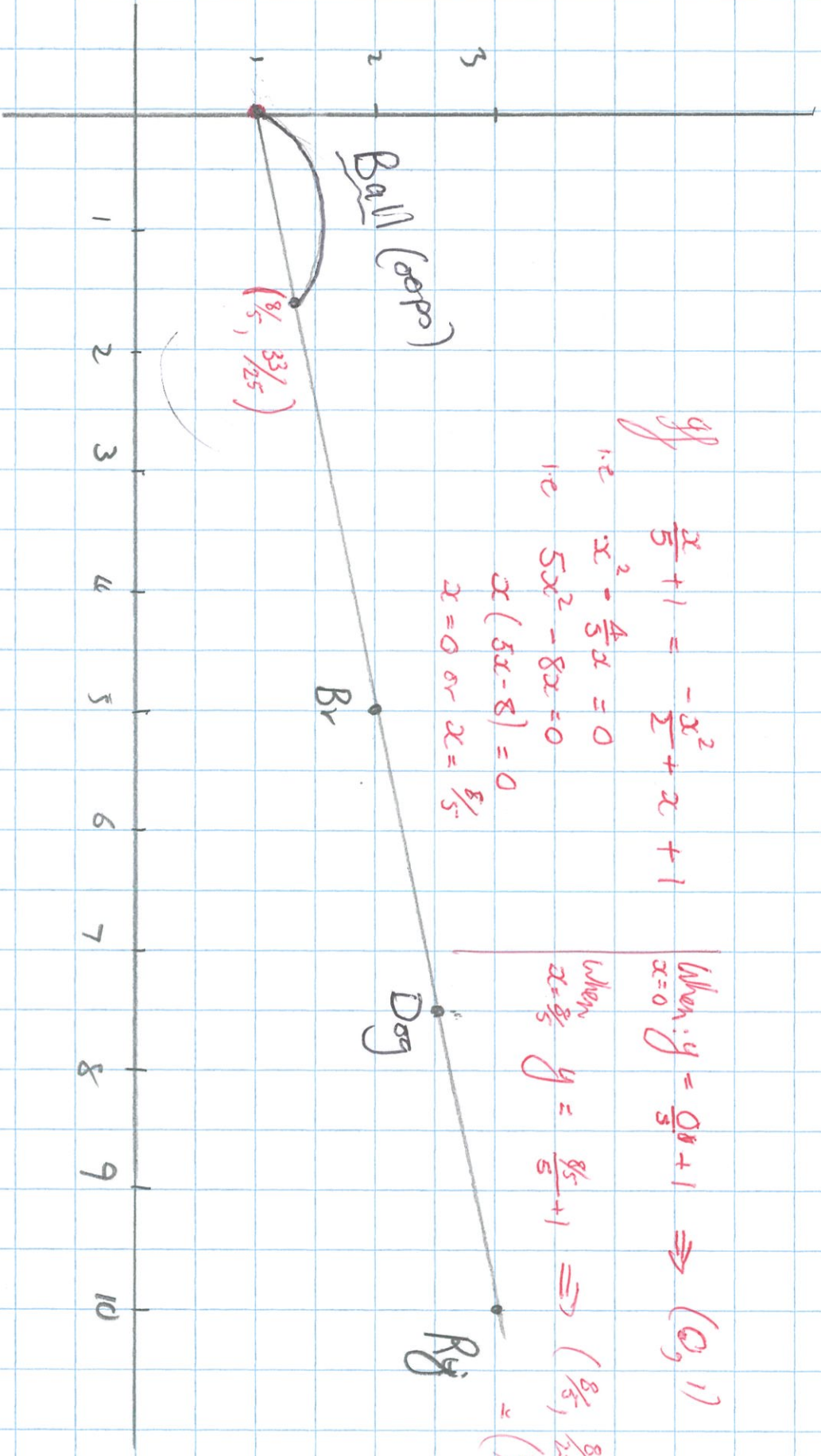
$y = \frac{x}{5} + 1 = \frac{-x^2}{2} + x + 1$

i.e $x^2 - \frac{4}{5}x = 0$

i.e $5x^2 - 8x = 0$

$x(5x - 8) = 0$

$x = 0$ or $x = \frac{8}{5}$

When $x=0$: $y = \frac{0}{5} + 1$ $\Rightarrow$ $(0, 1)$
When $x = \frac{8}{5}$ $y = \frac{8/5}{5} + 1$ $\Rightarrow$ $(\frac{8}{5}, \frac{8}{25} + 1)$ $(\frac{8}{5}, \frac{33}{25})$

Pay Both Correct

Question 3   **(3 marks)**

Find in terms of p the remainder when $x^3 - 2x^2 + px - 6$ is divided by $x - 2$.

$$x - 2 \overline{\smash{)}\begin{array}{r} x^2 + p \\ x^3 - 2x^2 + px - 6 \\ \end{array}}$$

$$- x^3 - 2x^2 \quad \checkmark_\circ$$

$$- \quad \begin{array}{r} px - 6 \\ px - 2p \end{array} \quad \checkmark_\circ$$

$$- 6 + 2p$$

$\therefore$ The remainder is $\underline{2p - 6}$ $\checkmark$

Question 4   **(3 marks)**

Points M(p,6) and N(1,-2) are the end points of line segment MN. If the midpoint of line is (-3, q). Determine the value of p and q.

(-7,6)

(-3,2)

-7   -3

4   N(1,-2)

$$\frac{p + 1}{2} = -3 \quad \checkmark_\circ$$

$$p + 1 = -6$$

$$p = -7 \quad \checkmark_\circ$$

$$\frac{6 + -2}{2} = q$$

$$\therefore \quad q = 2 \quad \checkmark$$

MATHEMATICS METHODS 1 & 2   **CALCULATOR-FREE**
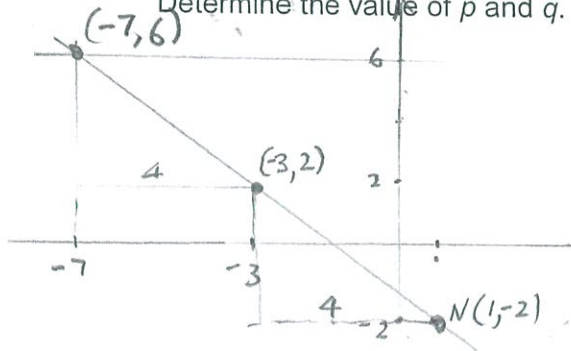
Question 5   **(6 marks)**

The function $y = f(x)$ is shown below.

-1 for Incorrect intercept

$(-2.5, 4)$

$(2, -2)$
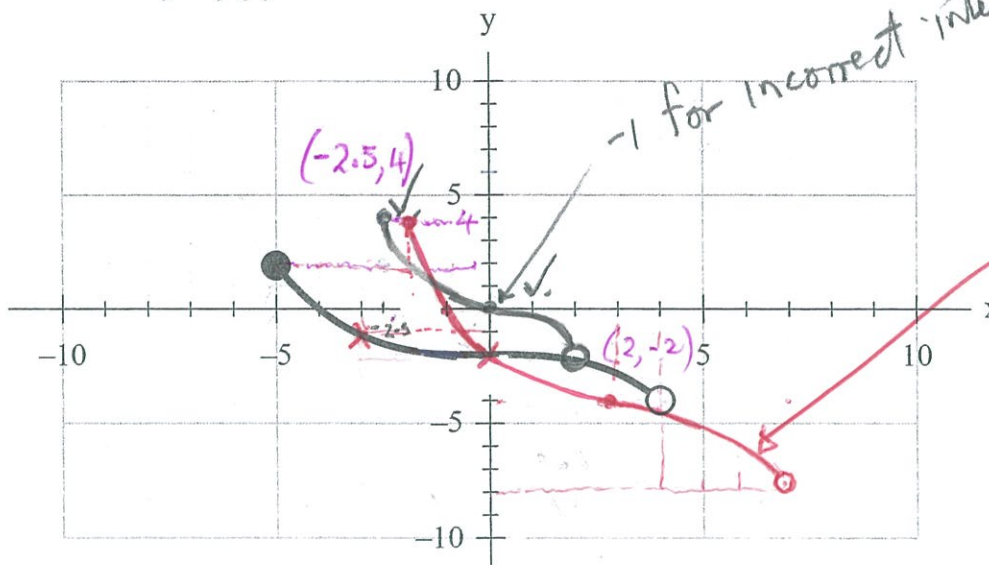
$g(x) = 2f(x-3)$

(a)   State the range of $f(x)$.   (1 mark)

$$\{f(x): \mathbb{R} : -4 < f(x) \leq 2\}$$

(b)   Another function is given by $g(x) = 2f(x - 3)$.

(i)   Describe the transformation required to produce $g(x)$ from $f(x)$.   (2 marks)

Any Order ✓ (i) Translation 3 to the Right followed
         ✓ (ii) Dilation scale factor ×2 // to y-axi

(ii)   State the coordinate of the y-intercept under this transformation.
                                                                      (1 mark)

$(0, -2)$ ✓

(c)   On the same axes above, sketch the graph of $y = f(2x) + 2$.   (2 marks)

see sketch

Question 6   **(7 marks)**

Given the equation $2^{x+1} + 2^{3-x} = 17$, $x \in \mathbb{R}$,

(a)   use a suitable substitution to rewrite the equation above as a quadratic equation. (3 marks)

$2^x \times 2^1 + 2^3 \times 2^{-x} = 17$

$2(2^x) + \frac{8}{2^x} = 17$ ✓

Let $y = 2^x$ ✓

$2y + \frac{8}{y} = 17$

$2y^2 + 8 = 17y$ ✓

$2y^2 - 17y + 8 = 0$

☑ Correctly rearrange the equation using $2^{-x} = \frac{1}{2^x}$
☑ Assign a suitable substitution
☑ Rewrite as a quadratic equation

(b)   hence, determine the solution(s) to the exponential equation.   (4 marks)

$2y^2 - 17y + 8 = 0$

$(2y - 1)(y - 8) = 0$ ✓

$y = \frac{1}{2}$ or $y = 8$ ✓

$2^x = \frac{1}{2}$ or $2^x = 8$

$x = -1$ or $x = 3$ ✓✓
☑☑ Correctly solve the quadratic equation from part a)

☑☑ Find both solutions to the <u>ORIGINAL</u> exponential equation.

IF STUDENTS DID NOT FIND THE CORRECT QUADRATIC EQUATION IN PART A:
- 1 mark ONLY for correctly solving a SIMPLE quadratic.
- A maximum of 2 marks for correctly solving a more COMPLEX quadratic correctly.
- 2 marks ONLY for the correct answer with no correct working. (Eg. Using substitution and trial and error)

MATHEMATICS METHODS 1 & 2

CALCULATOR-FREE

Question 7 (6 marks)

Should be $-3 < x \leq 0$

Given $f(x) = ax(x^2 + 4x + 3)$, $\quad -3 < f(x) \leq 0$ $\quad$ and $f(1) = -24$

(a) Rewrite the function in factorised form, stating the value of a. (3 marks)

$$-24 = a(1)(1^2 + 4(1) + 3)$$

space ↗

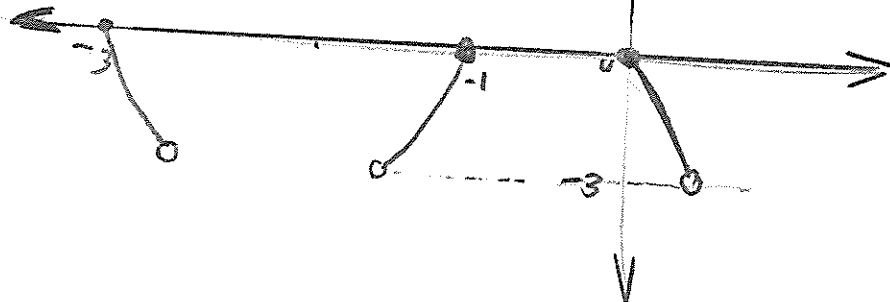(b) Sketch the function on the given domain. (3 marks)

$$-24 = 8a$$

$$a = -3 \checkmark$$

$$\therefore f(x) = -3x(x+3)(x+1) \quad (-2,3)$$

f(x)
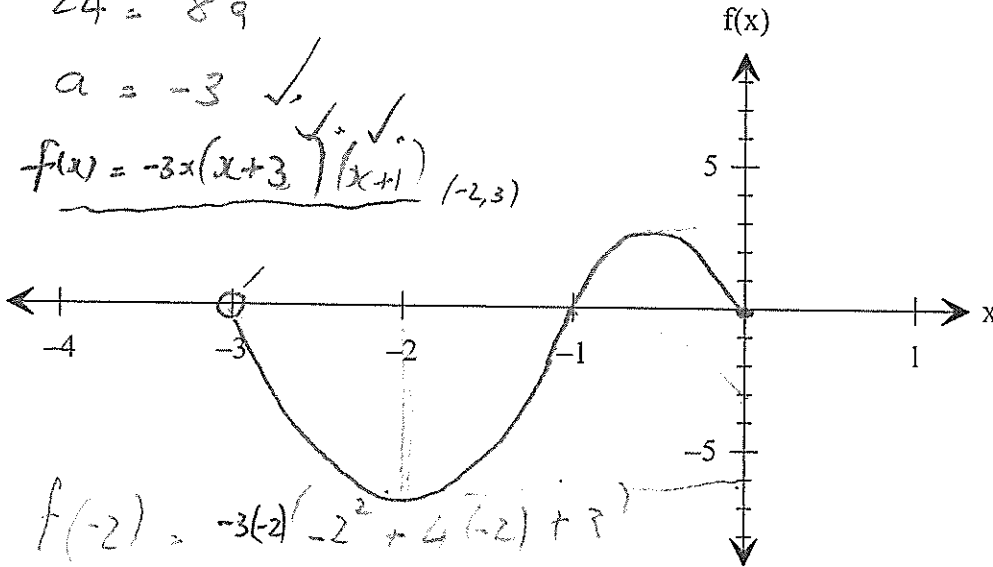
$f(-2) = -3(-2)(-2^2 + 4(-2) + 3)$

$= 6(-1)$

$= -6$

Correct Domain ✓
3 Roots ✓
Shape ✓

If they interpreting $-3 < f(x) \leq 0$ RANGE

Question 8

(6 marks)

A relationship between x and y is given by $x = \frac{7}{2}y + 2$.

(a)   Determine y when $x = 23$.

(2 marks)

$$23 = \frac{7}{2}y + 2$$
$$\frac{21 \times 2}{7} = y \checkmark$$
$$\therefore y = 6 \checkmark$$

(b)   State, with justification, whether x is a function of y.

(2 marks)

$\underline{x \text{ is a function}}$ of y because it is $\underline{a \text{ one to one}}$ relationship $\checkmark$

(c)   State, with justification, whether x is proportional to y.

(2 marks)

$\underline{x \text{ is not proportional to } y}$ as it does not go thru the Origin $\checkmark$

Question 9

(4 marks)

The quantity of light, Q, given out by a lamp is proportional to the square of the current, I, passing through the lamp. When $Q = 1000$, $I = 2$.

(a)   Write an equation that relates Q and I.

(2 marks)

$$Q = kI^2 \qquad k = \frac{1000}{2^2}$$
$$\underline{Q = 250I^2} \checkmark \qquad = 250 \checkmark$$

(b)   Find the value of Q when $I = 3$.

(1 mark)

$$Q = 250 \times 9$$
$$= 2250 \checkmark$$

(c)   What happens to Q when I is doubled.

(1 mark)

Q increases by a factor of $2^2 = \underline{4} \checkmark$